feuille TD5 exo 1 $\nearrow$ 1

$$f_\alpha(x,y) = -3x^2 - 3y^2 - 3\alpha^2 + 2xy + 2\alpha x + 2\alpha y$$

1/ $f_\alpha$ admet un seul point critique sur $\mathbb{R}^2$

Soit $M_1$ un pt critique $M_1 : (x_1, y_1)$ alors $\begin{cases} \frac{\partial f_\alpha}{\partial x}(M_1) = 0 \\ \frac{\partial f}{\partial y}(M_1) = 0 \end{cases}$ et réciproquement

Calculons : $\frac{\partial f_\alpha}{\partial x}(M) = -6x_1 + 2y_1 + 2\alpha$.

$\frac{\partial f_\alpha}{\partial y}(M) = -6y_1 + 2x_1 + 2\alpha$

d'où $M_1(x_1, y_1)$ pt critique si et seulement si $\begin{cases} -6x_1 + 2y_1 + 2\alpha = 0 \\ -6y_1 + 2x_1 + 2\alpha = 0 \end{cases} \Leftrightarrow \begin{cases} -6x_1 + 2y_1 = -2\alpha \\ 2x_1 - 6y_1 = -2\alpha. \end{cases}$

$\frac{L_1}{2} \begin{cases} -3x_1 + y_1 = -\alpha \\ x_1 - 3y_1 = -\alpha \end{cases}$ $\frac{L_2}{2}$ $\begin{cases} -3x_1 + y_1 = -\alpha \\ 3x_1 - 9y_1 = -3\alpha \end{cases}$ $\times 3 L_2$ $L_2 + L_1 \begin{cases} -3x_1 + y_1 = -\alpha \\ -8y_1 = -4\alpha \end{cases}$

$\begin{cases} -3x_1 + y_1 = -\alpha \\ +y_1 = +\frac{\alpha}{2} \end{cases}$ $\begin{cases} -3x_1 = -\frac{\alpha}{2} - \alpha \\ y_1 = \frac{\alpha}{2} \end{cases}$ $\begin{cases} -3x_1 = -\frac{3\alpha}{2} \\ y_1 = \frac{\alpha}{2} \end{cases}$ $\begin{cases} x_1 = \frac{\alpha}{2} \\ y_1 = \frac{\alpha}{2} \end{cases}$

Nous avons qu'une solution $(\frac{\alpha}{2}, \frac{\alpha}{2})$ unique.
ainsi il y a un seul pt critique $M_1(\frac{\alpha}{2}, \frac{\alpha}{2})$

2/ $f_\alpha(\frac{\alpha}{2}, \frac{\alpha}{2}) = -3\frac{\alpha^2}{4} - 3\frac{\alpha^2}{4} - 3\alpha^2 + \frac{2\alpha^2}{4} + \frac{2\alpha^2}{2} + \frac{2\alpha^2}{2} = -\frac{4\alpha^2}{4} - \alpha^2 = -2\alpha^2$

remarquons $f_\alpha(0,0) = -3\alpha^2$

3/ Mise sous forme de somme de carrés à coefficient négatif ou positif de $f_\alpha$

$$f_\alpha(x,y) = -3x^2 - 3y^2 - 3\alpha^2 + 2xy + 2\alpha x + 2\alpha y$$

$$-\frac{1}{3}f_\alpha(x,y) = +x^2 + y^2 + \alpha^2 - \frac{2}{3}xy - \frac{2}{3}\alpha x - \frac{2}{3}\alpha y$$

$$= x^2 - 2x\left(\frac{y}{3} + \frac{\alpha}{3}\right) + \left(\frac{y}{3} + \frac{\alpha}{3}\right)^2 - \left(\frac{y}{3} + \frac{\alpha}{3}\right)^2 + y^2 + \alpha^2 - \frac{2}{3}\alpha y$$

$$= \left(x - \left(\frac{y}{3} + \frac{\alpha}{3}\right)\right)^2 - \frac{y^2}{9} - \frac{\alpha^2}{9} - \frac{2\alpha y}{9} + y^2 + \alpha^2 - \frac{2}{3}\alpha y$$

$$= \left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 + \frac{8y^2}{9} - \frac{8\alpha y}{9} + \frac{8\alpha^2}{9} = \left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 + \frac{8}{9}\left[y^2 - 2y\frac{\alpha}{2} + \alpha^2\right]$$

$$= \left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 + \frac{8}{9}\left[\left(y - 2y\frac{\alpha}{2} + \frac{\alpha^2}{4}\right) - \frac{\alpha^2}{4} + \alpha^2\right] = \left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 + \frac{8}{9}\left[\left(y - \frac{\alpha}{2}\right)^2 + \frac{3\alpha^2}{4}\right]$$

$$= \left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 + \frac{8}{9}\left(y - \frac{\alpha}{2}\right)^2 + \frac{2}{3}\alpha^2$$

$$f_\alpha(x,y) = -3\left(x - \frac{y}{3} - \frac{\alpha}{3}\right)^2 - \frac{8}{3}\left(y - \frac{\alpha}{2}\right)^2 - 2\alpha^2$$

Vérification . 1/ $f_\alpha(0,0) =$
. 2/ $f_\alpha(\frac{\alpha}{2}, \frac{\alpha}{2}) =$.
. 3/ on redéveloppe.

feuille TD5 exo 1 suite

4- on remarque que quelquesoit $(x,y) \in \mathbb{R}^2$ $f_\alpha(x,y) \leq 0$

car somme de carrés à coefficient negatif

5- le point critique est alors un maximum.