feuille 4 exo XI

ln fonct° croissante ↗

1/ $\begin{cases} 4\ln\left(\frac{6}{p}\right) \geq 0 \\ 4\ln(2p-1) \geq 0 \end{cases} \iff \begin{cases} \ln\left(\frac{6}{p}\right) \geq 0 \\ \ln(2p-1) \geq 0 \end{cases} \iff \begin{cases} \frac{6}{p} \geq 1 \\ 2p-1 \geq 1 \end{cases} \iff \begin{cases} 6 \geq p \\ p \geq 1 \end{cases}$

$\iff 1 \leq p \leq 6$

2/ offre = demande : $\iff D(p) = S(p)$

$\iff 4\ln\left(\frac{6}{p}\right) = 4\ln(2p-1)$ ln fonct° ↗ stricte

$\iff \frac{6}{p} = 2p-1$

$\iff 6 = 2p^2 - p$

$\iff 2p^2 - p - 6 = 0$

$\iff (p-2)(2p+3) = 0$

$\iff \begin{cases} p = 2 \\ \cancel{p = -\frac{3}{2}} \end{cases}$ NON $\quad p_e = 2$

2b/ Quantité commune : $q_e(2) = 4\ln\left(\frac{6}{2}\right) = 4\ln(3) =$

3/ $D = 4\ln\left(\frac{6}{p}\right) \iff D = \ln\left[\left(\frac{6}{p}\right)^4\right] \iff e^D = \left(\frac{6}{p}\right)^4$

car $y = \ln x \iff x = e^y$.

$\iff e^D = \frac{6^4}{p^4} \iff p^4 = \frac{6^4}{e^D} \quad p = \frac{6}{e^{D/4}} \quad f(D) = \frac{6}{e^{D/4}}$

4/ $S = 4\ln(2p-1) \iff S = \ln\left[(2p-1)^4\right] \iff e^S = (2p-1)^4$

$\iff \frac{S}{4} = \ln(2p-1) \iff e^{S/4} = 2p-1 \iff p = \frac{e^{S/4}+1}{2} = g(S)$

$g(S) = \frac{e^{S/4}+1}{2}$

4/ $R = \int_0^{q_e} f(x)\,dx - p_e q_e = \int_0^{q_e} \frac{6}{e^{x/4}}\,dx - p_e q_e = \int_0^{q_e} 6 \cdot e^{-\frac{x}{4}}\,dx - p_e q_e$

$= 6\left[-4e^{-\frac{x}{4}}\right]_0^{q_e} - p_e q_e = 6\left[4 - 4e^{-\frac{q_e}{4}}\right] - p_e q_e = 24\left(1 - e^{-\frac{q_e}{4}}\right) - p_e q_e$

$D = 4\ln\left(\frac{6}{p}\right)$

$D' = 4 \times \left(-\frac{6}{p^2}\right) \frac{1}{\left(\frac{6}{p}\right)} = -24 \times \frac{p}{p^2 \times 6} = -\frac{4}{p}$ $\quad D' < 0$ pour $p \in [1;6]$

$S = 4\ln(2p-1)$

$S' = 4 \times \frac{2}{2p-1} = \frac{8}{2p-1}$ $\quad S' > 0$ pour $2p-1 > 0$ soit $p > \frac{1}{2}$

donc $S' > 0$ pour $p \in [1;6]$

p	1	2	6
D	$4\ln(6)$	$4\ln(3)$ 4,39	0
S	0	$4\ln(3)$ 4,39	$4\ln 11 = 9{,}59$

$4\ln(6) = 7{,}17$
$4\ln(3) = 4{,}39$
$4\ln(11) = 9{,}59$

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D	0	$4\ln(3)$	$4(\ln(6))$
p	6	2	1

$p = \frac{6}{e^{D/4}}$

S	0	$4\ln 3$	$4\ln 11$
p	1	2	1

$p = \frac{e^{S/4}+1}{2}$