feuille 4 exo VI suite :

$$I_4 = \int_1^e \frac{\ln x}{x^2}\,dx = \int u'v = uv - \int v'u \quad \text{avec} \begin{cases} u' = \frac{1}{x^2} & u = -\frac{1}{x} \\ v = \ln(x) & v' = \frac{1}{x} \end{cases}$$

$$I_4 = \left[-\frac{\ln(x)}{x}\right]_1^e - \int_1^e -\frac{1}{x^2}\,dx = \left(-\frac{1}{e}\right) + \left[\frac{1}{x}\right]_1^e = -\frac{1}{e} - \left(\frac{1}{e} - 1\right) = 1 - \frac{2}{e}$$

feuille 4 exo VII

1/ $C_m(q) = \frac{4q}{\sqrt{1+2q^2}}$ $\quad C(q) = \int \frac{4q}{\sqrt{1+2q^2}}\,dq$ avec $C(0) = 20$

$C(q) = \int \frac{u'}{\sqrt{u}}$ avec $u = 1+2q^2$ $u' = 4q$ $\quad C(q) = [2u^{1/2} + K]$

$C(q) = 2\sqrt{1+2q^2} + K$ $\quad C(0) = 20 = 2 + K$ $\quad K = 18$

$C(q) = 2\sqrt{1+2q^2} + 18$

2/ $R_m(q) = \frac{q}{1+q^2}$ $\quad R_m(q) = \frac{u'}{2u}$ ~~$\frac{u'}{u} = l$~~ avec $u = 1+q^2$ $u' = 2q$

$\int \frac{u'}{2u} = \frac{\ln(u)}{2}$ d'où $R(q) = \frac{1}{2}\ln(1+q^2) = \ln\left(\sqrt{1+q^2}\right)$

vérification $R'(q) = \frac{1}{2} \times \frac{2q}{1+q^2}$ $\quad \left(\ln\left(\sqrt{1+q^2}\right)\right)' = \frac{2q}{2\sqrt{1+q^2}} \times \frac{1}{\sqrt{1+q^2}}$

3/ $C_m(q) = (6q-1)e^{3q^2-q}$ avec $C(0) = 5$

$C_m(q) = u'e^u$ avec $u = 3q^2 - q$ $\quad u' = 6q - 1$

$\int u'e^u = e^u$ $\quad C(q) = e^{3q^2-q} + K$

$C(0) = 1 + K = 5$ $\quad K = 4$

$C(q) = e^{3q^2-q} + 4$

4/ $P_m(L) = \frac{1}{3L+1}$ $\quad P_m(L) = \frac{u'}{3u}$ avec $u = 3L+1$ $\quad u' = 3$

$\int \frac{u'}{3u} = \frac{1}{3}\ln(u)$ car $\left(\frac{1}{3}\ln(u)\right)' = \frac{1}{3}\frac{u'}{u} = \frac{3}{3(3L+1)} = \frac{1}{3L+1}$

$P(L) = \frac{1}{3}\ln(3L+1)$

VIII