

Feuille 1 exo VII

$U[\alpha, \beta]$ $\beta > \alpha$, $\int_{\alpha}^{\beta} k dx = 1 \Rightarrow k(\beta - \alpha) = 1 \quad k = \frac{1}{\beta - \alpha}$

$E(X) = \frac{\beta + \alpha}{2}$ ~~et comme~~ $E(X) = 1$ alors $\beta = 2 - \alpha$. car: $E(X) = \int_{\alpha}^{\beta} \frac{x}{\beta - \alpha} dx = \left[\frac{x^2}{2} \right]_{\alpha}^{\beta} \frac{1}{\beta - \alpha}$

$V(X) = 3$ et comme $V(X) = \frac{(\alpha - \beta)^2}{12}$ car $E(X^2) = \int_{\alpha}^{\beta} \frac{x^2}{\beta - \alpha} dx = \frac{\beta^3 + \alpha\beta + 2\alpha^2}{3}$

et $(E(X))^2 = \left(\frac{\beta + \alpha}{2}\right)^2$

ainsi on a le systeme: $\begin{cases} \beta + \alpha = 2 \\ \frac{(\alpha - \beta)^2}{12} = 3 \end{cases}$ $\begin{cases} \beta + \alpha = 2 \\ (\alpha - \beta)^2 = 36 \end{cases}$ $\begin{cases} \beta + \alpha = 2 \\ |\alpha - \beta| = 6 \end{cases}$

soit $\begin{cases} \beta + \alpha = 2 \\ \beta - \alpha = 6 \end{cases}$ ou $\begin{cases} \beta + \alpha = 2 \\ \beta - \alpha = -6 \end{cases}$

soit: $\beta = 4$ et $\alpha = -2$, soit $\beta = -2$ et $\alpha = 4$.

et comme $\beta > \alpha$ alors $U[-2; 4]$